

Space-time as a lattice of harmonic oscillators: quantitatively deriving relativity, mass, and gravitation

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Received _____; accepted _____

ABSTRACT

The microscopic structure of space and time is investigated. It is proposed that space and time of an inertial observer Σ are most conveniently described as a crystal lattice Λ , with nodes representing measurement ‘tickmarks’ and connected by independent quantized harmonic oscillators which vibrate more severely the faster Σ moves with respect to the object being measured (as due to the Uncertainty Principle). The Lorentz transformation of Special Relativity is to be derived. Further, mass is understood as a localized region $\Delta\Lambda$ having higher vibration temperature than that of the ambient lattice. The effect of relativistic mass increase may then be calculated without appealing to energy-momentum conservation. The origin of gravitation is shown to be simply a transport of energy from the boundary of $\Delta\Lambda$ outwards by lattice phonon conduction, as the system tends towards equilibrium. Application to a single point mass leads readily to the Schwarzschild metric, while a new solution is available for two point masses - a situation where General Relativity is too complicated to work with. The important consequence is that inertial observers who move at relative speeds too close to c are no longer linked by the Lorentz transformation, because the lattice of the ‘moving’ observer has already disintegrated into a liquid state.

In this work I propose a model for the microscopic structure of space and time to understand relativity and the origin of mass and gravitation in terms of statistical mechanics.

The underlying ideas originated from an examination of Special Relativity (Einstein

1905), which does not treat distances and times as absolute. For a uniformly ‘moving’ observer, one can disregard spatial dimensions perpendicular to the velocity of motion \mathbf{v} and elaborate the previous statement as meaning that if Σ' measures: (i) position differences between events at the same time, or (ii) time differences at the same position, the resulting distances and intervals are smaller than those of a ‘stationary’ observer Σ who measures the same events from a frame at rest with respect to them, by a common factor $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$. Since each of the two effects (i) and (ii) was presented without interference from the other, they can in general be superposed to form the Lorentz transformation (i.e. rotation) of space-time (Lorentz 1923). One could also describe the phenomenon as due to a measuring apparatus having different intrinsic properties when set in ‘motion’. Thus another way of expressing (i) is to visualize the ‘tickmarks’ which calibrate the ruler of Σ' as more widely separated than those of Σ , as illustrated in Figure 1. If a unit of length for Σ is x_m then the same for Σ' will be $x_m\gamma$, which is of the form $x_m\sqrt{N}$ ($N \geq 1$), suggesting that microscopically space-time might be undergoing a ‘random walk’ process of some kind. Similarly, (ii) may also be viewed in this way, because the Lorentz transformation of time is the same as that of space, except time is being re-scaled by the speed $c = 1/\sqrt{\epsilon_o\mu_o}$ to form a different dimension, see Figure 2. To avoid duplication, the ensuing treatment will emphasize effect (i) only.

The particular manner of considering Special Relativity as changes in the measuring devices of a travelling observer matches well with a statistical approach: an obvious way of further separating the ‘tickmarks’ of a ‘moving’ ruler, is if they are not part of a rigid body, but their relative positions can fluctuate while in motion, maintaining the mean translational velocities $= \mathbf{v}$. I shall pursue the fluctuations specifically as oscillations, the amplitude of which increase with the relative speed v . The spirit of approach is in accordance with the Heisenberg Uncertainty Principle (Heisenberg 1930): the higher the velocity (or momentum) being measured the larger the inherent uncertainties (deviations)

in the ability of an apparatus to determine p and x , with the product $\Delta p \Delta x$ reaching the smallest value $\sim \hbar$ only for ground states where motion of the object is minimized. I therefore let the natural separation of the ‘tickmarks’ be x_o (note that $x_o < x_m$, for reasons to be explained). To this I add a sinusoidal variable x_1 , which has a mean $\langle x_1 \rangle = 0$. The total separation $x = x_o + x_1$, then, has a mean $\langle x \rangle = x_o$, but the r.m.s. value is given by:

$$x_{rms} = x_o \left(1 + \frac{\langle x_1^2 \rangle}{x_o^2} \right)^{\frac{1}{2}} \quad (1)$$

and is $> x_o$. For the purpose of distance measurements it is the magnitude of x , not the sign, which calibrates the length between a pair of ‘tickmarks’ at any phase of the cycle. If x oscillates rapidly¹ and substantially about x_o , it is then x_{rms} which defines a unit of length. An analogy is that of our mains AC power supply. Most electric appliances recognize only the magnitude of V and have response times slow compared with typical AC cycle frequencies of \sim a few $\times 10$ Hz, so that the relevant output voltage is V_{rms} , even though $\langle V \rangle = 0$.

The fundamental postulates of the theory are stated here. The starting point concerns the first two of them.

(a) Space *and* time form a crystal lattice, the lattice points (hereafter simply referred to as ‘nodes’, which represent the ‘tickmarks’ of an observer) are connected by independent harmonic oscillators with quantized energy levels $E = (n + 1/2)\epsilon$. All inertial observers Σ at rest relative to each other are assigned a common rest frame lattice Λ with which

¹Caution should be exercised while interpreting this word, because not only do the spatial ‘tickmarks’ oscillate, but the same applies to time which behaves just like space (see earlier). Thus the period of a cycle cannot be defined with respect to ordinary time. The quantum mechanical approach of this paper alleviates somewhat the urgency of settling the issue, however, since one considers $\langle x^2 \rangle$ as an expectation value, proportional to energy.

measurements are naturally performed.

(b) The degree of oscillation of Λ is parametrized by a temperature which may be defined not only with respect to Σ , but also *any* inertial observer Σ' . If $\Sigma' = \Sigma$, this will become the rest temperature, which is taken to have an ambient value of 0 K. If a relative speed $v \neq 0$ exists between the two frames, this temperature will be > 0 K, by an amount T which increases with v in a simple manner to be formulated below; as a result the quantity x_{rms} is larger for Σ' than for Σ .

(c) Matter owes its origin to a localized region $\Delta\Lambda$ of Λ having a rest temperature > 0 K. The quantity we define as rest *mass* is proportional to this surplus of energy within $\Delta\Lambda$. An oscillator in $\Delta\Lambda$ has larger than ambient amplitude, i.e. $x_{rms} \rightarrow X_{rms}$. With respect to Σ' this amplitude is further increased, by the same factor as x_{rms} does in Postulate (b), see Figure 3. The situation may be likened to Σ being already in a local Lorentz frame (which widens x_{rms} to X_{rms}), and Σ' moving relative to Σ at the stipulated speed v .

(d) The effect of gravitation is due to conduction of energy by the oscillator waves (phonons) from the hotter region of $\Delta\Lambda$ to the cooler ambient region. The process causes a gradual downward trend in the temperature as one moves away from $\Delta\Lambda$. There is consequently a distribution of oscillator lengths, thereby affecting the separation of the measurement ‘tickmarks’, see Figure 3. This is the reason for the curvature of space-time responsible for the existence of universal gravitation in the lattice surrounding $\Delta\Lambda$.

None of the above is in conflict with the First Relativity Postulate, viz. that no preferred frame of rest exists. There is also no violation of the Second Postulate.

I now focus on Special Relativity, viz. Postulates (a) and (b). When applying to inertial observers in relative motion, note that even for Σ the ambient separation between nodes of Λ is larger than x_o , due to the zero point energy which leads to a finite $\langle x_1^2 \rangle$

given by $\langle x_1^2 \rangle = E(n=0)/\kappa = \epsilon/2\kappa$ where κ is the oscillator (spring) constant. The absolute minimum unit of length is then obtained from Equ (1) as

$$x_m = x_o \left(1 + \frac{\epsilon}{2\kappa x_o^2} \right)^{\frac{1}{2}} \quad (2)$$

Since energy is additive, for finite v (ambient $T > 0$) x_m increases to:

$$x_{rms} = x_m \left(1 + \frac{\langle x_1^2 \rangle}{x_m^2} \right)^{\frac{1}{2}} \quad (3)$$

where $\langle x_1^2 \rangle = \bar{E}(T)/\kappa$ is the mean energy at temperature T , calculated with the lowest level now having energy $E = \epsilon$ (i.e. $n = 1$). The relevant Partition Function is $Z = 1/[1 - \exp(-\epsilon/kT)]$, leading to a mean energy of $\bar{E} = \epsilon Z e^{-\frac{\epsilon}{kT}}$. Substituting these into Equ (3), one obtains:

$$x_{rms} = x_m \left(1 + \frac{\epsilon}{2\kappa x_o^2} \frac{e^{-\frac{\epsilon}{kT}}}{1 - e^{-\frac{\epsilon}{kT}}} \right)^{\frac{1}{2}} \quad (4)$$

At this point I complete postulate (b) above with the following quantitative relationship between v and T :

$$\frac{v^2}{c^2} = e^{-\frac{\epsilon}{kT}} \quad \left(= \frac{\bar{E}}{\epsilon + \bar{E}} \right) \quad (5)$$

Combining (4) and (5), one gets:

$$x_{rms} = x_m \left(1 + \frac{\epsilon}{2\kappa x_o^2} \frac{\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} \right)^{\frac{1}{2}} \quad (6)$$

Finally, postulate (a) is made more specific by invoking:

$$\epsilon = 2\kappa x_o^2, \quad (7)$$

which simply states that the zero point fluctuations double the mean-square separation of nodes from the natural value of x_o^2 . Eqs (4), (5) and (6) altogether tell us that the unit length for observer Σ' is not x_m , but $x_{rms} = x_m \sqrt{1/(1 - v^2/c^2)}$, implying smaller measured distances for Σ' than those for Σ , in accordance with Lorentz contraction.

There remains the possibility that the success achieved so far is an illusion, viz. statistical mechanics of harmonic oscillators is a wrong and completely irrelevant approach to questions concerning the nature of space and time. Although it so happens that Special Relativity can be explained in this way, the procedure could be merely formal: a wide variety of mathematical formulae describe the many diverse phenomena of the known physical world, the fact that some of the formulae resemble each other in appearance does not immediately imply a parallel in the physics involved. In this instance, however, such concerns are settled by the theory's ability to go beyond Special Relativity, to shed light on the issues of mass and gravitation.

I first discuss the relativity of mass. According to Postulate (c), mass is an energized lattice region $\Delta\Lambda$ wherein an oscillator has large r.m.s. amplitude, as illustrated in Figure 2. Upon transformation from Σ to Σ' (so that Λ is no longer the rest lattice) X , like x_m , is required by (c) and (b) to increase by a factor γ . Thus the oscillator energy is increased by γ^2 . The number of oscillators is, of course, decreased by γ , see Figure 2. As a result, the total energy within $\Delta\Lambda$ is higher with respect to Σ' by γ . This then affords an exceedingly simple derivation of the relativistic increase of the energy within $\Delta\Lambda$. By (c) the same effect applies to mass.

Next, I demonstrate how equally straightforward it is to explain gravitation as an energy transport effect. Let us center $\Delta\Lambda$ at the origin of the rest lattice Λ , and consider the 1-D conduction of energy in the +x-direction from some position $x = x_{min}$, Figure 3. Note that the notion of an inner boundary from which conduction commences is an abstract one: the boundary does not have to represent the physical size of the mass depicted in Figure . The transport equation is:

$$\sigma_{th} \frac{dT}{dx} = n\bar{E}\bar{v}. \quad (8)$$

Here $\sigma_{th} = n\bar{v}\lambda d\bar{E}/dT$ is the thermal conductivity of phonons: n is the linear phonon

density, \bar{v} is the ‘speed of sound’ in the lattice², $\bar{E}(T)$ is the mean energy of a phonon, and λ is the phonon mean free path, which is the size of the available lattice (since phonons do not interact), i.e. $\lambda = x$. Thus equ (8), together with the meaning of the various symbols involved, imply that

$$-x \frac{d\bar{E}}{dx} = \bar{E}, \quad \text{or} \quad \bar{E} = \frac{1}{\alpha x} \quad (9)$$

where α is a constant of integration. Combining equs (5) and (9), one obtains

$$\frac{v^2}{c^2} = \frac{1}{\alpha \epsilon x + 1} \quad (10)$$

as equation giving the speed of the local Lorentz (or ‘free fall’) frame at x .

Since the x -axis can represent a radial direction, we now write $x = r - r_g$ and $r_g = x_{min}$. In the limit of $r \gg r_g$ ($v \ll c$) there should be agreement with Newtonian gravity. This requires $\alpha \epsilon = c^2/2GM$, which removes the arbitrariness of the solution. Returning to equ (9), one clearly sees that $\bar{E} \propto M$, consistent with the proposition in Postulate (c) that mass is proportional to incumbent energy. At high speeds the role of r_g must be taken into account ($v \rightarrow c$ as $r \rightarrow r_g$). In the case of spherical symmetry there is only one free parameter in the problem, i.e. r_g must depend on α . By setting $r_g = 1/\alpha \epsilon = 2GM/c^2$, equ (10) reduces to $v^2/c^2 = r_g/r$, or $\gamma = (1 - \frac{r_g}{r})^{-\frac{1}{2}}$. It is therefore apparent that the ‘tickmarks’ of r (or x) are non-uniformly distributed throughout Λ , due to the energy outflow, in such a way that if they are used to measure r (or x) the result is radius in Euclidean (flat) space-time. In fact, the expression for γ contains all the information one needs to construct the full Schwarzschild metric (Schwarzschild 1916) of General Relativity. For example, as a falling object approaches the gravitational radius (or event horizon) $r = r_g$ time dilation

²The conduction of energy takes place in Λ , which is a lattice of space *and* time. Thus, like the oscillations, there is the need to introduce a new ‘time’ axis when defining the propagation speed of these phonons - signature of a fifth dimension.

becomes infinite. The notion of a ‘boundary’ for $\Delta\Lambda$ is now clear: phonon energy transport to other parts of the lattice takes place only beyond the event horizon.

Apart from its ability to offer insightful, even elegant, explanations of fundamental laws of physics, the model presented here can also make predictions. One immediate application is the problem of two point masses, where no known solution of the Einstein Field Equations exist. The phonon conduction approach reduces the mathematical complexities by manifold. A member of the pair may be taken as the mass M described above (except now $M \rightarrow M_1$) while the other (mass M_2) is positioned at Euclidean distance R from the first. The separation between their event horizons is then $R - r_g - r'_g$, where $r_g = 2GM_1/c^2$ and $r'_g = 2GM_2/c^2$. Since the presence of mass M_2 cannot alter the boundary conditions for the outward energy transport from mass M_1 ³ and vice versa, one simply solves for the oscillator (phonon) energy E_1 due to mass M_1 , likewise E_2 due to M_2 , for any position along the line joining the two masses and distance r from the first. The total energy at this position is $\bar{E} = \bar{E}_1 + \bar{E}_2 = \epsilon[r_g/(r - r_g) + r'_g/(R - r - r'_g)]$. By using equ (5), we have the speed v of the local Lorentz frame at r :

$$\frac{v^2}{c^2} = \frac{\frac{r_g}{r-r_g} + \frac{r'_g}{R-r-r'_g}}{1 + \frac{r_g}{r-r_g} + \frac{r'_g}{R-r-r'_g}} \quad (11)$$

This gives for the first time the space-time metric at any position between two masses. For more complicated mass distributions, the metric at any point is given by the superposition of all the outwardly conducted energy distributions, in the same manner as above.

In conclusion, I found that a simple microscopic model of space-time can explain the Lorentz transformation and the origin of mass and gravitation very naturally. In this model, space and time form a crystal lattice, with nodes connected by harmonic oscillators.

³Otherwise one will not be able to restore the single mass solution by letting the other mass tend to zero

The notion of distances and times has no meaning unless measurements are performed using the nodes as ‘tickmarks’ to define unit intervals. The faster the speed of an object, the ‘fuzzier’ the measurement, and this is reflected in equ (5) wherein the nodes vibrate with a temperature T which increases with the relative speed v between a rest lattice and a ‘moving’ object. When $v = c$, T reaches infinity, implying that the lattice has already disintegrated before that. Thus there must exist a threshold v ($< c$) which corresponds to a critical T , observers who move with respect to each other at speeds high than this value are no longer connected by the Lorentz transformation. In other words, while Relativity links the macroscopic properties of crystal lattices at different vibration temperatures, here we are comparing two fundamentally different lattices, viz. those of a crystal and a liquid.

I am indebted to Dr Massimiliano Bonamente for suggesting a harmonic oscillator model for the space-time lattice of a moving observer.

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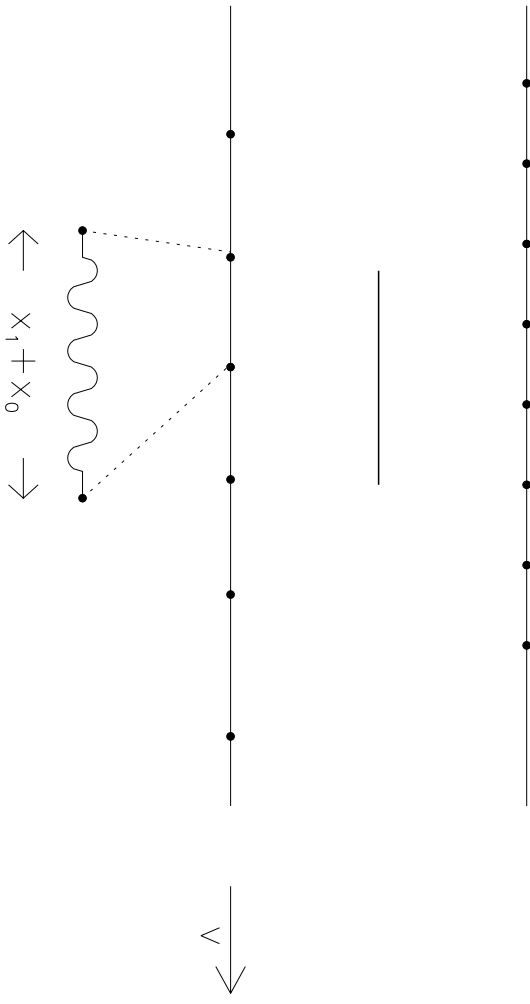
Figure Captions

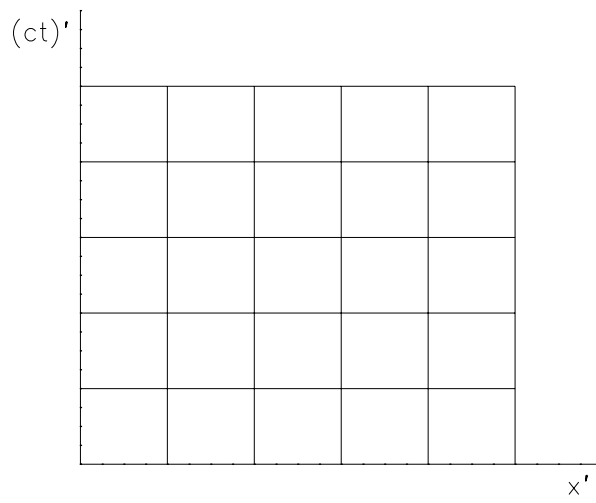
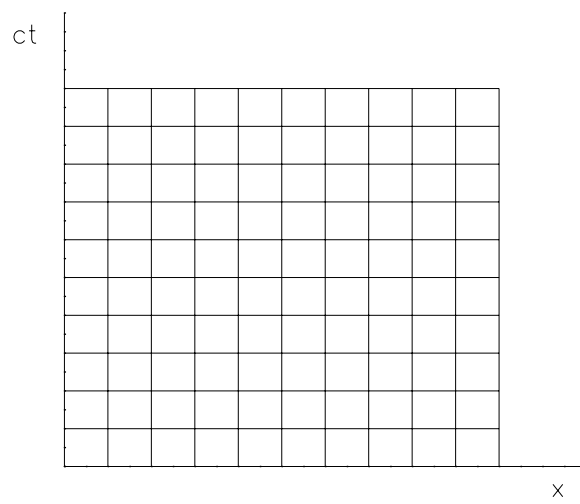
Figure 1: The space-time lattice of stationary (Σ) and moving (Σ') observers are illustrated here for the case of distance measurements. The ‘tickmarks’ of the ruler of Σ are marked as the topmost set of black dots. The rod to be measured is the short bar immediately beneath, and is at rest with respect to Σ . Observer Σ' measures the length of this rod while in motion, by simultaneously acquiring data on the positions of the front and rear end of the rod. It is postulated that effectively Σ' is using a moving set of ‘tickmarks’, and if microscopically these are connected by oscillators which vibrate while in motion, the ‘tickmarks’ widen as depicted in the lower half of the diagram. Consequently Σ' obtains a smaller value for the length of the rod.

Figure 2: The space-time lattices of inertial observers Σ (top) and Σ' (bottom), the latter ‘moving’ at velocity \mathbf{v} with respect to the former, who is regarded as ‘stationary’. If Σ' measures Δx between two ‘stationary’ events at the same time, or Δt at the same position, in each case the result is less than that obtained by Σ . This means for ‘orthogonal’ measurements of space and time performed by Σ' , the ‘tickmarks’ of Σ are more widely separated, as indicated by the lower grid.

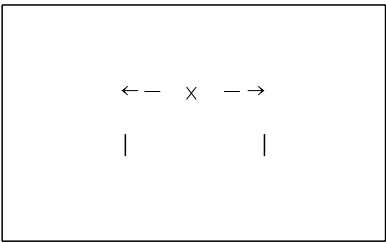
Figure 3: Illustrating the origin of mass and gravitation. *Top:* a region $\Delta\Lambda$ of Λ (the rest frame lattice of observer Σ) has higher than ambient temperature. Mass is proportional to the incumbent extra energy. The physical boundary of the mass (shown here in the space dimension only) is drawn as a rectangular box, inside of which all the energy surplus resides, as a result the nodes are much more widely spaced than outside. *Middle:* The same region as it appears in the lattice of observer Σ' who ‘moves’ with respect to Σ at velocity \mathbf{v} . Separation between any pair of nodes is now increased by the factor γ , meaning fewer oscillators within $\Delta\Sigma$, but more energy per oscillator. The net increase in enclosed energy (hence mass) is γ (see text). *Bottom:* Energy is conducted outwards from $\Delta\Lambda$ to the

ambient lattice by phonons, which causes the separation between nodes to gradually reach the natural minimum at asymptotically large distances. Note that the inner boundary from which the energy transport commences is an abstract quantity which need not be the physical size of the mass; in fact, the former is usually within the latter.



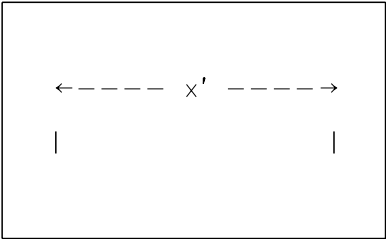


$$\Sigma: \quad | \quad | \quad | \quad |$$



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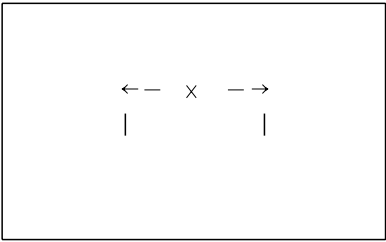
$$\Sigma': \quad \overset{\text{-----} \rightarrow \vec{v}}{\quad} | \quad | \quad |$$



$$| \quad | \quad | \quad |$$

$$-\vec{v} \leftarrow \text{-----}$$

$$\Sigma: \quad | \quad | \quad | \quad | \quad | \quad | \quad |$$



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